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A Note on Brans-Dicke Cosmology with Axion

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We study the Brans-Dicke model in the presence of an axion. The dynamical equations are solved when the fields are space independent and the metric is spatially flat. It is found that at late time the scale factor undergoes decelerated expansion but the dilaton grows large. At early time, scale factor and the dilaton approach constants.

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1. Exploring the implications of duality symmetry in string theory, the possibility of having inflation without relying on the potential energy density is pointed out in [1], [2], [3]. However, exiting from inflationary period turned out to be a problem essentially because of the presence of singularities [3]. Two possible directions have been proposed since then. First one is to investigate the higher order corrections in string action which might become strong to smooth out the singularity [4], [5], [6]. The second is the possibility of exiting from the inflationary period via quantum tunneling through classically forbidden region [7] , [8].

Besides graviton and dilaton, string theory predicts a three rank tensor field(H) in the massless spectrum of the theory. In [9], [10], [11], [12], [13] cosmological evolutions have been studied is presence of such field. In particular, in [11], a general analytic solution has been given for flat Friedmann-Robertson-Walker(FRW) metric. It was shown that the presence of this H field significantly modifies the evolution of the metric and the dilaton. It was found that the very presence of axion field may prevent the universe from a singular collapse.

On the other hand, recently in [14], exploiting a possible scale factor duality symmetry, Brans-Dicke(BD) theory in presence of the cosmological constant has been analyzed. Here, in the action the dilaton kinetic energy term carries a free parameter ω instead of usual $\omega = -1$ that appears in the low energy string theory(for more on these theories, even when ω is a dilaton dependent function, see the paper [15] and references there in). If we also introduce a three rank tensor field in such model, the action takes the form

$$S = \int d^4x \sqrt{-g} e^{-\phi} (R - \omega \partial_\mu \phi \partial^\mu \phi - \frac{1}{12} H^2), \tag{1}$$

where ϕ is the dilaton field. It is not very clear if this model with $\omega \neq -1$ can arise in string theory but let us however note the following. String theory in four dimension will certainly contain many moduli fields. The detail of their number and structure follows from the detail of the compactification scheme. Nevertheless, if it so happens that one of the moduli fields has a flat direction along the dilaton, one might expect to have ω different from -1. At the same time, however, even if this action is not derivable from string theory, it may be worthwhile to study it on its own right as an interesting model for gravity theories. At this stage, we would like to mention that BD cosmology has also been studied [16] in the presence of a particular class of massless matter where the massless field does not couple to the dilaton in the BD frame (unlike the action (1) here). These models

may arise in string theory where the field strength (analogue of H here) comes from the Ramond-Ramond sector rather than the Neveu-Schwarz sector as it is in our case.

In what follows, we study (1) in a flat FRW background.

2. We take the metric of the form:

$$dS^2 = -dt^2 + e^{2\alpha(t)} dx^i dx^j \delta_{ij}. \tag{2}$$

Provided all the fields are independent of the space coordinates, the action (1) can be brought to the following form:

$$S = \int dt e^{(3\alpha-\phi)} \{-6(\partial_t \alpha)^2 + 6\partial_t\alpha\partial_t\phi + \omega(\partial_t\phi)^2 - \frac{q^2}{2}e^{-6\alpha}\}. \tag{3}$$

The constant q is related to the three rank tensor fields [13] as follows: the dual of $H_{\mu\nu\rho}$ is an axion (Θ)in four dimension which is defined as

$$H^{\mu\nu\rho} = e^{\phi}\epsilon^{\mu\nu\rho\lambda}\partial_\lambda\Theta. \tag{4}$$

Since the action does not have any potential energy contribution for the axion field, the momentum follows from the above lagrangian is constant. So we have defined $p_\Theta = \frac{\partial\mathcal{L}}{\partial\partial_t\Theta} = q$.

The constraint equation and the equations of motion are:

$$\omega\partial_t^2\phi + 3(1+\omega)(\frac{\partial_t a}{a})\partial_t\phi + 3(\frac{\partial_t^2 a}{a}) = 0, \tag{5}$$

$$6(\frac{\partial_t a}{a})^2 - 6(\frac{\partial_t a}{a})\partial_t\phi - \omega(\partial_t\phi)^2 - \frac{q^2}{2}e^{-6\alpha} = 0, \tag{6}$$

$$\omega\partial_t^2\phi - 6(\frac{\partial_t a}{a})^2 + 3(\omega+2)(\frac{\partial_t a}{a})\partial_t\phi + \frac{3q^2}{2(2\omega+3)}e^{-6\alpha} = 0. \tag{7}$$

where the scale factor $a = e^{\alpha}$.

Fortunately (5) - (7) can be solved exactly for generic $\omega(> -\frac{3}{2})$. The solutions are (the detail is given in the appendix)

$$\phi = -\frac{1}{2}\ln[\frac{c(2\omega+3)}{4q^2}\mathrm{sech}^2\frac{\sqrt{c}\tau}{2}], \tag{8}$$

$$a = [\frac{4q^2}{c(2\omega+3)}\cosh^2\frac{\sqrt{c}\tau}{2}]^{\frac{1}{4}} e^{\frac{1}{4}\sqrt{\frac{c(2\omega+3)}{3}}\tau}, \tag{9}$$

where the dilaton-time τ is defined as

$$t = \int e^{-\bar{\phi}} d\tau \tag{10}$$

with

$$\bar{\phi} = \frac{1}{4}[\ln\{\frac{c(2\omega+3)}{4q^2}\text{sech}^2\frac{\sqrt{c}\tau}{2}\} - \sqrt{3c(2\omega+3)}\tau]. \tag{11}$$

Here c is an arbitrary constant. All other constants that appear from integrating (5) - (7) are set to zero for convenience. Since it will not play an important role in our discussion, for simplicity we will also set $c = \frac{4q^2}{(2\omega+3)}$. Now using (10) , one can express (8) , (9) in terms of cosmic time t. Though, (10) is hard to compute exactly, it can be integrated numerically. The behaviour is shown in fig.1 for $\omega = 1$ and $q^2 = 5$. On the other hand, it is easy to extract out early and late time behaviour of the scale factor and the dilaton. It turns out that for $\tau \to \infty$(or $t \to \infty$),

$$e^{\phi} \sim t^{\frac{2}{1+\sqrt{3(2\omega+3)}}}, \tag{12}$$

$$a \sim t^p, \quad p = \frac{1+\sqrt{\frac{2\omega+3}{3}}}{1+\sqrt{3(2\omega+3)}}, \quad \text{with } \frac{da}{dt} > 0, \quad \frac{d^2a}{dt^2} < 0. \tag{13}$$

So we see that the scale factor undergoes a decelerated expansion similar to the post-big bang regime as in [1], [2]. However, the theory becomes strongly coupled at late time as the original coupling constant e^{ϕ} gets large. It is easy to check that for $\omega = -1$, we recover the behaviour of [10], [11]. Furthermore, when $\omega \to \infty$, we have $a \sim t^{\frac{1}{3}}$ and e^{ϕ} goes to a constant. When $\tau \to 0(t \to 0)$, to first order in t

$$e^{\phi} \sim \text{constant} + \mathcal{O}(t^2), \tag{14}$$

$$a \sim 1 + \frac{q}{2\sqrt{3}}t + \mathcal{O}(t^2). \tag{15}$$

In fig.2 and fig.3, we have ploted the scale factor and the dilaton as functions of time for $\omega = 1$ and $q^2 = 5$. So we notice that at late time, the effect of axion on the graviton-dilaton system is practically nil. However, at early time the axion seems to have significant influence on the behaviour of the universe.

To this end, we would also like to mention that, there is another set of solutions of (5) - (7) which would correspond to the $t < 0$ branch. Those are given by:

$$\phi = -\frac{1}{2}\ln[\frac{c(2\omega+3)}{4q^2}\text{sech}^2\frac{\sqrt{c}\tau}{2}] \tag{16}$$

$$a = [\frac{4q^2}{c(2\omega+3)}\cosh^2\frac{\sqrt{c}\tau}{2}]^{\frac{1}{4}}e^{-\frac{1}{4}\sqrt{\frac{c(2\omega+3)}{3}}\tau} \tag{17}$$

and

$$\bar{\phi} = \frac{1}{4}[\ln\{\frac{c(2\omega+3)}{4q^2}\mathrm{sech}^2\frac{\sqrt{c}\tau}{2}\} + \sqrt{3c(2\omega+3)}\tau] \tag{18}$$

with t and τ are realated as in (10) .

3. Appendix.

Here we briefly mention one way to solve (5) - (7) .

Define $\bar{\alpha}$ and $\bar{\phi}$(the one introduced in the text to define dialton-time τ) as

$$\alpha = A\bar{\alpha} + B\bar{\phi}, \tag{19}$$

$$\phi = C\bar{\alpha} + D\bar{\phi}, \tag{20}$$

where A, B, C, D are

$$A = \frac{1}{3\omega+4}\sqrt{\frac{2\omega+3}{3}}, \quad B = -\frac{1+\omega}{3\omega+4} \tag{21}$$

$$C = \frac{\sqrt{3}}{3\omega+4}\sqrt{\frac{2\omega+3}{3}}, \quad D = \frac{1}{3\omega+4}. \tag{22}$$

Further, defining

$$Y = 3A\bar{\alpha} + D\bar{\phi}, \quad X = D\bar{\alpha} + 3A\bar{\phi}, \tag{23}$$

the equations can be reduced to

$$Y'' - \frac{q^2}{(2\omega+3)}e^{-2Y} = 0, \quad X'' = 0, \tag{24}$$

along with the constraint equation

$$(Y')^2 - (X')^2 + \frac{q^2}{2\omega+3}e^{-2Y} = 0. \tag{25}$$

Here, in the above equations, prime denotes differentiation with respect to τ. Now, note that the first equation of (24) is just the Liouville like equation which can easily be integrated. By suitably choosing the constant of integrations arising from (24) , the constraint equation (25) can be made to satisfy.

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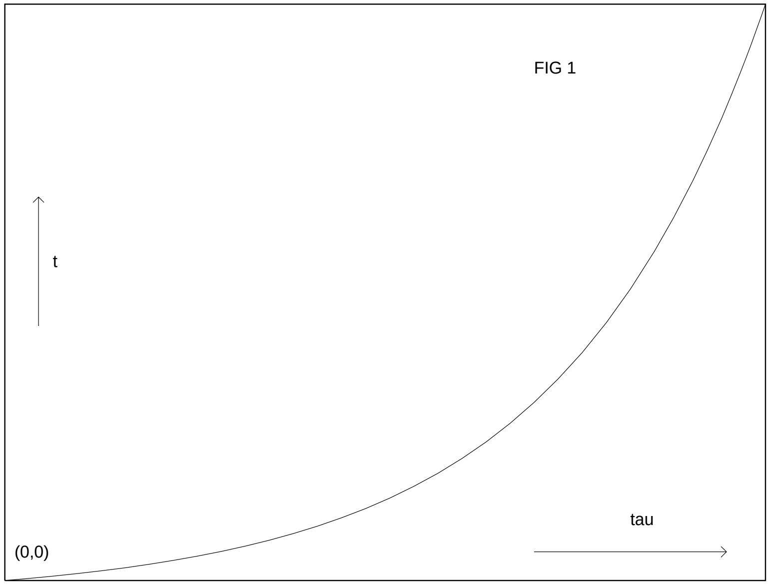

FIG 1

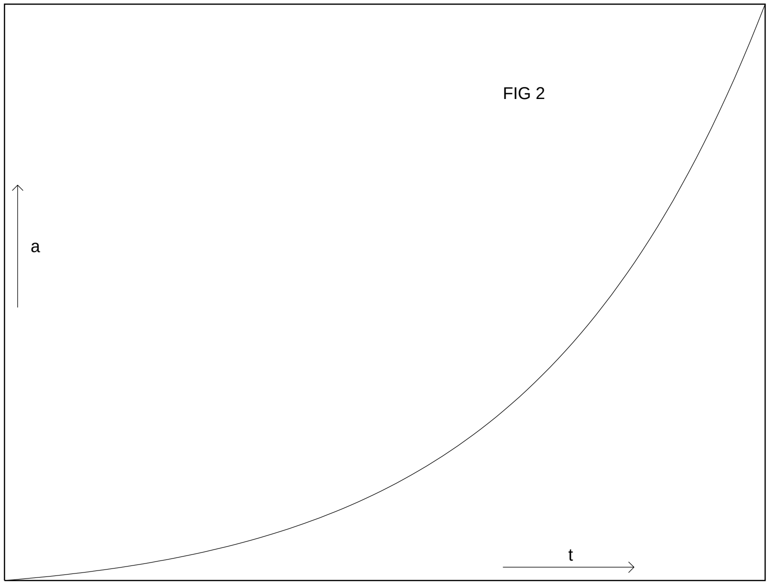

FIG 2

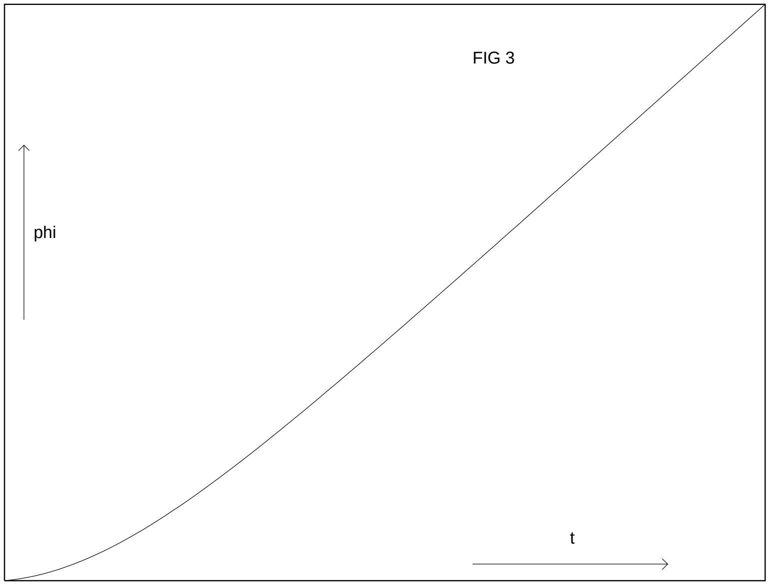

FIG 3